Progressive Education Society's
Modern College of Arts, Science & Commerce Ganeshkhind, Pune – 16
End Semester Examination: Jan.2023
Faculty: Science and Technology

Program: BscGen03 **Semester:I** **SET: B**
Program (Specific):Mathematics **Course Type:Core Type**
Class:F.Y.Bsc.(Regular)
Name of the Course:Calculus-I **Max Marks:35**
Course Code:22-MT-112 **Time: 2Hr**
Paper: II

Instructions to the candidate:

1) ***There are 4 sections in the question paper. Write each section on separate page.***
2) ***All Sections are compulsory.***
3) ***Figures to the right indicate full marks.***
4) ***Draw a well labelled diagram wherever necessary.***

SECTION: A

Q1) Choose the correct alternative: **[Marks 5]**

1. Which of the following subset of R is not bounded :
   a) $\{1, 2, 3, ..., 100\}$ b) N
   c) $[-1, 1]$ d) (-1,1)

2. The monotonically increasing sequence is convergent if and only if it is _____
   a) bounded above b) bounded below
   c) unbounded d) oscillating

3. The sequence $\left\{\sin sin\left(\frac{n\pi}{2}\right)\right\}, n \in N$ is _____
   a) convergent b) bounded
   c) increasing d) decreasing

4. The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if _____
   a) $p > 1$ b) $0 < p < 1$
   c) $p = 1$ d) all $p \in R$

5. The function $f(x) = x^3$ is _____
   a) even function b) odd function
   c) periodic function d) neither even not odd function

Q2) Attempt any four : **[Marks 4]**

1. Solve the inequality: $|4x - 5| \leq 13$.
2. Define monotonically increasing sequence, monotonically decreasing sequence with one example of each .
3. Discuss the convergence of the following sequence : $\{1, 2, 1, 2, 1, 2, ...\}$.
4. Find $\left(1 + \frac{1}{n}\right)^{3n}$.
5. State Comparison test for convergence of series.
6. Find the domain and range of the function: $f(x) = \frac{1}{\sqrt{2x-7}}$.

SECTION: B

Q3) Attempt any four : **[Marks 8]**

1. Prove that $|a + b| \leq |a| + |b|$, $\forall\, a, b \in R$.
2. Find the supremum and infimum of the following set $S = \{1 - \frac{1}{n} \mid n \in N\}$.
3. Find : $\frac{n^{10}}{10^n}$.
4. Disccuss the convergence of the following series :
   a) $\sum_{n=1}^{\infty} \frac{\cos cos\,(2n)}{n^3}$ b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.
5. Discuss for what values of p the series converges: $\sum_{n=1}^{\infty} \frac{1}{n(logn)^p}$
6. Sketch the graph of following function:
   a) $f(x) = x^2$, $x \in [-1, 1]$
   b) $f(x) = \sin sin\, x$, $x \in R$

SECTION: C

Q4) Attempt any Four: **[Marks 8]**

1. Show that if $a \neq 0$, $b \in R$, are such that $a.b = 1$ then, $b = \frac{1}{a}$.
2. Find the rational number between $\sqrt{5}$ and $\sqrt{6}$.

3. The first few terms of the sequence are given below ,find the formula for n^{th} term of the following sequences :

a) $\left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots\right\}$ b) $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots\right\}$.

4. Discuss the convergence of the series : $\sqrt{\frac{1}{4}} + \sqrt{\frac{2}{6}} + \ldots + \sqrt{\frac{n}{2(n+1)}}$

5. Determine whether the mapping f: $N \to N$ defined by $f(x) = \frac{x}{2}$ is a function or not

Justify !

6. Define least integer function and find $\lceil 2.6 \rceil, \lceil -1.5 \rceil$

SECTION: D

Q5) Attempt any two : **[Marks 10]**

1. Find all $x \in R$ that satisfy the following inequality: $|3x + 4| < |x + 2|$
2. Show that every convergent sequence is bounded . Is the converse true ?
3. Show that the following sequences are divergent

a) $\left\{1 + (-1)^n\right\}$ b) $\left\{\sin sin \frac{n\pi}{4}\right\}$.

4. Using Cauchy condensation test, show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent